much he may be impressed by the mathematical ingenuity of the investigation.

Postscript.

A discussion followed the reading of this paper in which it was pointed out that there exist motions in the solar system which cannot be wholly accounted for by gravitation acting in accordance with Laplace's Differential Equation; and it was suggested that this indicates that Laplace's equation requires modification, at least for bodies in relative motion. Now if Laplace's equation has to be modified it appears certain from the physical interpretation of Mr. Whittaker's equations that the modification cannot be that substitution of equation (2) for Laplace's equation which is suggested by Mr. Whittaker; and that what we learn from the investigation is that we may limit the inquiry by dismissing this alternative when searching for the modification.

But has it been quite ascertained that the observed motions may not be due to causes already known? We are acquainted with at least three causes acting within the solar system which *must* produce deviations from Newtonian motions. The chief of these is the meteoric streams and sporadic meteorites which dash about in vast swarms within the space through which the visible bodies of the solar system have to travel. Another such cause is the escape of gases from atmospheres (see *Astrophysical Journal* for June 1900, Appendix, p. 369). This escape we have reason to believe takes place chiefly from the side of a planet turned towards the Sun, and therefore produces on the planet a one-sided effect; and a third cause which is known to exist is electric. Modern astronomers are able to compute perturbations with such refinement that it does not seem impossible that the first of the above *veræ causæ* may produce effects of sufficient amount to be detected.

It is noteworthy that at least the first two of these *veræ causæ* would operate with more intensity upon *Mercury* than upon our Earth: witness much other evidence, corroborated by the existence of such phenomena as the Solar Corona and the Zodiacal Light.

Note on the Present Condition of the Lunar Theory.

By E. Nevill.

1. The recent discussion at the March Meeting of the Royal Astronomical Society on Professor Newcomb's note, entitled "On the desirableness of a reinvestigation of the problems growing out of the Mean Motion of the Moon," suggests my furnishing some information with respect to the researches on

the motion of the Moon which have been carried out at the Natal Observatory.

2. First with respect to the comparison of the observed and tabular places of the Moon. The whole mass of observations contained in Professor Newcomb's "Reduction and Discussion of Observations of the Moon before 1750" (Washington, 1878); in Sir G. Airy's "Greenwich Lunar Reductions for 1750 to 1830," and its extension "The Greenwich Lunar Reductions for 1831 to 1851"; and the complete series of Greenwich Observations of the Moon made with the Transit Circle between 1851 and 1899, have all been reduced on a uniform system of star places and elements of reduction, to a homogeneous series of comparison between observation and Hansen's "Tables de la lune."

There are thus obtained a homogeneous complete series of comparison between observations and tables extending over the period 1630 to 1900. This may be called "Data A."

3. The preceding comparison between the observed and tabular places of the Moon is insufficient as it stands to furnish any explanation of the origin of the outstanding discrepancies whose existence it reveals. For this purpose what is required is to so combine the results of the comparison between observation and tables, that they will yield separately for each year—

(1) The coefficient expressing the mean error depending on the Moon's Mean Longitude.

(2) The coefficient expressing the mean errors depending on the sine and cosine of the Moon's *Mean Anomaly.*

(3) The coefficients expressing the mean errors depending on the sine and cosine of the *Variation.*

(4) The coefficients expressing the mean errors depending on the sine and cosine of the *Annual Equation.*

(5) The coefficients expressing the mean errors depending on the sine and cosine of the *Parallactic Inequality.*

(6) The coefficients expressing the mean errors depending on the sine and cosine of the *Evection.*

(7) The coefficients expressing the mean error of the Moon's Parallax and its variation.

(8) The coefficients expressing the mean error varying directly as the Moon's Latitude.

(9) The coefficients expressing the mean errors varying as the sine and cosine of the Argument of Latitude.

The available observations prior to 1750 are too few to enable these to be systematically calculated for each year, but for the one hundred and fifty years from 1750 to 1899, from the preceding comparison between Hansen's tabular place of the Moon and that as observed at Greenwich, there have been computed for each year the value of the preceding eighteen coefficients. In addition the more accurate modern places of the Moon obtained by means of the Greenwich Transit Circle since 1851, have been

utilised for computing the separate yearly values of a number of other coefficients depending on other auxiliary arguments.

The long series of yearly values of these different coefficients furnishes full data for the investigation from the observations of the origin of the outstanding imperfections of Hansen's Tables. They may be called "Data B."

4. These outstanding imperfections in Hansen's Tables may be divided into the following classes :—

(1) Errors in the theoretical values for the coefficients of the perturbations due to the direct action of the Sun employed in the theory embodied in Hansen's "Tables de la lune."

(2) Consequential errors due to Hansen having adopted erroneous values for the different constants depending on observation which are involved in his theory.

(3) Errors in the values assigned by Hansen to the perturbations due to the disturbing action of the planets, the figure of the Earth and similar causes, and omission of terms representing perturbations of sensible magnitude whose existence he had overlooked.

Considering the first of these classes, it may be taken that the values assigned by Hansen in the "Darlegung" to the ordinary lunar solar perturbations correspond very closely to the true theoretical values, if Hansen's values for the constants be adopted. There are few cases where the error in the coefficients exceeds some hundredth of a second of arc. Generally, they agree with the values found by Delaunay, when both are reduced to the same basis. In all cases where there exists any material discrepancy between the theoretical values of Hansen and Delaunay, the correct value of the coefficient has been recomputed, carrying the calculation when necessary to the eleventh power of m. The existing discrepancies arise mainly from Delaunay not having carried his computations far enough to obtain the complete value of the coefficient, but in some cases they are due to Delaunay having overlooked combinations of terms contributing to the complete value. In a very few cases Hansen's value seems incomplete, probably from the accidental omission of terms of high order. It is true that the values of these coefficients embodied in Hansen's Tables are not identical with the final values given in the "Darlegung," but the differences are comparatively unimportant, and no revision of theory will sensibly alter the great mass of theoretical values embodied in Hansen's Tables.

With respect to the second of these classes, the values assigned by Hansen to the constants which have to be derived from observation, nearly all require material amendment. This is especially the case with respect to the secular motion of the *Mean Longitude*, of the *Longitude of Perigee*, and of the *Longitude of Node*, as well as with the value assigned to the *Secular Accelera-*

tion, and the ratio of the parallaxes of the Sun and Moon. But in addition corrections are required to the values employed by Hansen for the constants of *Eccentricity* of *Inclination* and of *Parallax*.

The corrections to Hansen's tabular values of these constants have been derived from the complete series of values termed "Data B," and Hansen's Tables amended by supplying the requisite auxiliary tables necessary to take them into account. For the sake of completeness it has been necessary to do this on two separate systems.

1st. By supposing that Hansen's Tables are complete as they stand, and that after rectifying an admitted error, they merely require the removal of the admittedly incorrect empirical term depending on the argument (8 V − 13 E).

2nd. By admitting that Hansen's Tables are not complete, but after the removal of the empirical term, require supplementing by the addition of certain smaller terms due to the action of the planets, such as the *Jovian Evection*, of whose existence and approximate value there can be no question.

Yet the difference between the two resulting systems of corrections is far less than might have been imagined.

It is with the third class of imperfections in Hansen's Tables that the main difficulty arises. The values which Hansen determined from theory for the coefficients of the perturbations of this character, all need some correction, for in no case has he carried his computations far enough to obtain the complete value. Generally speaking, the difference is not large enough to be of importance, but in the case of the terms depending on the longitude of the Moon's Node, and the perturbation of *Venus*, there is a considerable difference between Hansen's value and the true values of the terms.

But the crucial point is: How many terms of this class having sensible magnitude are absent from Hansen's Tables? About half a dozen there can be no question, mainly the planetary evectional terms; but how many beyond these, and of what magnitude and what period? Therein lies the difficulty. Opinion varies.

5. The long series of values for each year of the coefficients of the principal inequalities in the motion of the Moon, which constitutes "Data B," can be made to show what are the inequalities which are not included in Hansen's Tables, and yet are necessary to reconcile the observed and tabular places of the Moon.

They have been employed for this purpose, and by the suitable discussion of the variations in the yearly values of the twenty different coefficients, there have been deduced the probable periods of the different outstanding terms, and the approximate values of their coefficients.

Some of these outstanding terms are found to correspond closely in both period and coefficient with those known to exist and to be absent from Hansen's Tables, and the fact that the values so determined correspond so closely with the values furnished by theory furnishes the strongest evidence of the reality of the existence of the other similar terms whose values are not so readily determined from theory.

6. The full details of the results of the reduction and discussion of this great mass of observations as compared with Hansen's Tables, has been lying for some time at the Natal Observatory awaiting publication. For it was decided by the Natal Government that as the work represented a considerable part of the scientific work done at the Colonial Observatory, it must not be published in any other form than as part of the publications of the colony. Up to the present no provision has been made to meet the cost of printing, but in the estimates for the coming year, it is trusted, provision will be made for the due printing and publication of this investigation. If so, they will appear in the course of the coming year, and the enormous mass of work be available for general use.

7. With regard to the theoretical calculation of these inequalities it is not necessary to say anything, as my views are well known. Most of those which require consideration have been calculated by two independent methods which have been already described in the *Monthly Notices* of the Society. So far the final reduction and numerical calculation has been suspended, so that the values deduced from the observations shall remain absolutely independent. The theoretical determination of these terms will form, therefore, a subsequent volume of the publications of the Natal Observatory as soon as the necessary funds for printing are available; and as it will be a heavy mass of mathematical printing, its cost will be much heavier than the portion of the complete whole formed by the reduction and discussion of the Greenwich lunar observations of the last 150 years.

Natal Observatory:
1903 *May* 9.

Note on the use of Peirce's Criterion for the Rejection of Doubtful Observations. By S. A. Saunder, M.A.

It would seem at first sight that when for observations made at the telescope we substitute a series of photographic measures we ought to be able to eliminate all those abnormal errors or real mistakes, such as the entry of a wrong figure, which are not con-